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Biped Robots: Effects of Small Perturbations on the Generation of Modular Trajectories [1]

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Abstract: Humanoid robots have been extensively studied in the last few years. The motivation for this study is that bipedal locomotion is superior to wheeled approaches on real terrain and situations where robots accompany or replace humans. Some examples are, on the development of human assisting device, such as prosthetics, orthotics, and devices for rehabilitation, rescue of wounded troops, maidens, accompany and assistance to elderly people, amongst others. Online generation of trajectories for these robots is a complex process, that includes different types of movements, i.e., distinct motor primitives. In this paper, we consider two motor primitives: rhythmic and discrete. We study the effect on a bipeds robots' gaits of inserting the discrete part as an offset of the rhythmic primitive, for synaptic and diffusive couplings. We also study stability of biped gaits. We simulate a periodic solution corresponding to the biped *run*, for the variation of the discrete offset. We find that amplitude and frequency of this periodic solution, are almost constant in all cases studied. This is useful when considering implementations of the proposed controllers for generating trajectories for the joints of real biped robots.

Keywords: Bipedal walking, Central Pattern Generator, Motor primitives, Changes in periodic solutions

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1 Introduction

There has been an increase interest in the study of animal and robot locomotion. Many models for the generation of locomotion patterns of different animals have been proposed [5, 12, 26]. The main goal is the understanding of the neural bases that are behind animal locomotion and then use this information to generate online trajectories on robots. In vertebrates, goal-directed locomotion is a complex task, involving the Central Pattern Generators (CPGs), located in the spinal cord, the brainstem command systems for locomotion, the control systems for steering and control of body orientation, and the neural structures responsible for the selection of motor primitives [13]. CPGs are networks of neurons that are responsible for the locomotion movements in $2n$- legged animals [24, 5, 12, 26]. Mathematically, CPGs are modeled by coupled nonlinear dynamical systems. CPGs are realized through silicon chips [31].

In Robotics, dynamical systems are a valuable tool for online generation of trajectories, since they allow their smooth modulation through simple changes in the parameter values of the systems of ordinary differential equations. Dynamical systems have also low computational cost, are robust against small perturbations, and allow phase-locking between the different oscillators [30, 7, 26, 19].

Taga *et al* [30] propose a new principle of sensorimotor control of legged locomotion in unpredictable environment. They use neurophysiological knowledge and nonlinear dynamical systems to implement the model. They achieve a stable and flexible locomotion, resulting from a global entrainment between the rhythmic activities of a nervous system, composed of coupled neural oscillators, and the rhythmic movements of a musculo-skeletal system. Feedback from the surrounding environment is also included. Schöner *et al* [29] propose a set of organizational principles that allow an autonomous vehicle to perform stable planning. Dégallier *et al* [7] use a dynamical systems' approach yielding the online generation trajectory in a robot performing a drumming task. These trajectories have both rhythmic and discrete parts. In 2001, Paul [23] include morphological parameters in the optimization of biped robot locomotion. Moreover, they consider three sets of experiments in which couple control and morphological parameters, the later concerning mass distribution, for the evolution of stable bipedal gaits. Authors show how mechanical design decisions and controller optimization can be accomplished in a single step and can lead to more mutually optimized systems. Nakanishi *et al* [21] use movement primitives, modeled by nonlinear oscillators, to implement learning in biped locomotion. The main goal was to design a controller that enabled natural human-like locomotion in a biped robot. The trajectories were learned through movement primitives by locally weighted regression, and their frequency was adjusted by a frequency adaptation algorithm based on phase resetting and entrainment of coupled oscillators. Geng *et al* [10] built a biologically inspired controller for a biped robot, that includes local reflexes and does not employ any kind of position or trajectory-tracking control algorithm. This controller allows the robot to exploit its own natural walking dynamics. Parameters of the reflexive controller are also tuned using a policy gradient reinforcement learning algorithm. Komatsu and Usui [15], study walk and run in a biped robot. They suggest a hybrid central pattern generator(H-CPG) method to realize adaptive dynamic motions including stepping and jumping. H-CPG are the usual CPG models in addition with the force control system, controlling the acting force from a leg to the floor, both in vertically and horizontally directions. Nakamura *et al* [20] present a reinforcement learning method allowing a biped robot not only to walk stably, but also to adapt to environmental changes. Matos *et al* [19] propose a bio-inspired robotic controller able to generate locomotion and to easily switch between different types of gaits.

In this paper, we develop the work by Pinto *et al* [27]. We assume a modular generation of a biped robot movements, supported by current neurological and human motor control findings [2, 13]. Our study is based in the work by Pinto and Golubitsky [26]. We consider the CPG model biped-robot (Figure 1) for biped robots' movements, which has the same architecture as a CPG for biped animals' movements [26]. The main difference is that here each neuron/cell is considered a CPG-unit, composed of two motor primitives: rhythmic and discrete. Both primitives are modeled by nonlinear dynamical systems. The rhythmic primitive produces the periodic solutions identified with common animal gaits, whereas the discrete primitive inserts a perturbation in the rhythmic part of the movement. We study the variation in the amplitude and the frequency values of a periodic solution produced by the CPG model biped-robot when the discrete primitive is inserted as an offset of the rhythmic part. The goal is to show that these discrete corrections may be performed since that they do not affect the required amplitude and frequency of the resultant trajectories, nor the gait, in the cases studied here. We also study the stability of bipedal gaits. Amplitude and frequency may be identified, respectively, with the range of motion and the velocity of the robot's movements, when considering implementations of the proposed controllers for generating trajectories for the joints of real robots.

2 CPG for bipedal robots' locomotion

In this section, we review the work done by Pinto and Golubitsky [26] on the CPG model biped-robot. We write the general class of systems of ODEs that model CPG biped-robot resume the symmetry techniques that allow classification of periodic solutions produced by this CPG model and identified with biped locomotor patterns. We also study bifurcation and stability of primary bipedal gaits.

2.1 Architecture and symmetry

Figure 1 shows the CPG model biped-robot for generating locomotion for bipeds robots. It consists of four coupled CPG-units. The CPG-units (or cells) are denoted by circles and the arrows represent the couplings between cells. All cells are identical, since they are represented by the same symbol. There are two distinct arrows that represent two distinct coupling strengths. Cells labeled LL_i indicates left leg cells, and cells labeled RL_i refer to right leg cells, where $i = 1,\ 2$.

Each cell is a CPG unit and is divided into two motor primitives, discrete and rhythmic, both modeled by nonlinear dynamical systems.

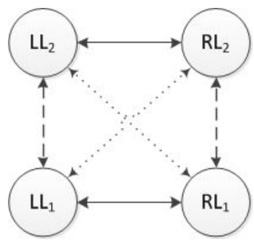

Figure 1: CPG biped-robot for biped robots locomotion. LL_i (left leg cells), RL_i (right leg cells), where $i = 1,\ 2$.

This is the minimal CPG capable of producing all of the bipedal gaits (see Table 1 in [26]). The choice of a 4-cells CPG model opposed to a 2-cells is explained by the phase shifts associated to gaits *walk* and *run*. *Walk* and *run* are two gaits in which left and right legs are half period out-of-phase. Nevertheless, *walk* and *run*are distinct gaits. Muscle groups in leg move as a pendula in the *walk*, and as a pogo stick in the *run* [17]. Thus, a CPG model with two neurons wouldn't suffice to predict these two gaits.

2.2 Ordinary differential equations of CPG biped-robot

The class of systems of differential equations of the CPG model for the biped model biped-robot is of the form:

$$\begin{aligned}
\dot{x}_{LL_1} &= F(x_{LL_1}, x_{RL_1}, x_{LL_2}, x_{RL_2})\\
\dot{x}_{RL_1} &= F(x_{RL_1}, x_{LL_1}, x_{RL_2}, x_{LL_2})\\
\dot{x}_{LL_2} &= F(x_{LL_2}, x_{RL_2}, x_{LL_1}, x_{RL_1})\\
\dot{x}_{RL_2} &= F(x_{RL_2}, x_{LL_2}, x_{RL_1}, x_{LL_1})
\end{aligned} \tag{1}$$

where $x_i \in \mathbf{R}^k$ are the cells i variables, where $i = LL_j,\ j = 1,2$, represent the two muscle groups in the left leg, analogously for $i = RL_j$. $k \geq 2$ is the dimension of the internal dynamics for each oscillator, and $F : (\mathbf{R}^k)^4 \to \mathbf{R}^k$ is an arbitrary mapping. The fact that the dynamics of each cell is modeled by the same function F indicates that the cells are assumed to be identical.

2.3 Symmetries of CPG biped-robot and bipedal gaits

Network biped-robot has

$$\Gamma_{\text{biped-robot}} = \mathbf{Z}_2(\omega) \times \mathbf{Z}_2(\kappa)$$

symmetry. biped-robot has the bilateral symmetry of animals ($\mathbf{Z}_2(\kappa)$), that allows for signals sent to the two legs to be interchanged. The translational symmetry ($\mathbf{Z}_2(\omega)$), forces the signals sent to the two cells in each leg to be the same, maybe up to a phase shift. Symmetry κ is given by $(LL_1RL_1)(LL_2RL_2)$, and symmetry ω is written as $(LL_1LL_2)\ (RL_1RL_2)$. $\mathbf{Z}_2$ is the cyclic group of order 2.

Theorem 3.6 (page 67) of Golubitsky and Stewart [11] classifies all possible symmetry types of periodic solutions for a given coupled cell network. These periodic solutions are then identified with bipedal rhythms. Let $x(t)$ be a solution of an ODE $\dot{x} = f(x)$, with period normalized to 1. Its periodic solutions are characterized by two symmetry groups: spatio-temporal symmetry group H and spatial symmetry group K. Spatio-temporal symmetries H fix the solution up to a phase shift, i.e., let $\gamma \in \Gamma$, then $\gamma x(t) = x(t-\theta) \leftrightarrow x(t+\theta) = x(t)$, where θ is the phase shift associated to γ. Spatial symmetries' action in the solution is trivial, i.e., $\gamma x(t) = x(t)$. If $\theta = 0$, then γ is a spatial symmetry. The pair (H,K) is a symmetry of a periodic solution $x(t)$ iff H/K is cyclic. There are twelve pairs of symmetry types (H,K) such that H/K is cyclic. In Table 1, we show, as an example four of those pairs, the corresponding periodic solutions and their identification with primary biped locomotor patterns. Primary gaits are gaits for which the spatiotemporal symmetry is the whole dihedral group $\Gamma_{\text{biped-robot}} \simeq \mathbf{D}_2$. For more information see [26]. In what follows, we explain how the identification of bipedal gaits is done.

H	K	**Left leg**	**Right leg**	**Gait**
$\Gamma_{\text{biped-robot}}$	$\Gamma_{\text{biped-robot}}$	(x_{LL},x_{LL})	(x_{LL},x_{LL})	*two-legged hop*
$\Gamma_{\text{biped-robot}}$	$\mathbf{Z}_2(\omega\kappa)$	(x_{LL},x^S_{LL})	(x^S_{LL},x_{LL})	*walk*
$\Gamma_{\text{biped-robot}}$	$\mathbf{Z}_2(\kappa)$	(x_{LL},x^S_{LL})	(x_{LL},x^S_{LL})	*two-legged jump*
$\Gamma_{\text{biped-robot}}$	$\mathbf{Z}_2(\omega)$	(x_{LL},x_{LL})	(x^S_{LL},x^S_{LL})	*run*

Table 1: Periodic solutions, and corresponding symmetry pairs, identified with primary bipedal gaits, where period of solutions is normalized to 1. ^{S} is half period out of phase.

Farley *et al* [9] discuss high and low frequency hops. In high frequency hopping, the motion is similar to a spring-mass system (or pogo stick-like motion). At low frequencies, the *two-legged jump* is a gait where ground contact is longer and appears to require ankle rotation, as in the *walk*. CPG biped-robot predicts two hops, the *two-legged hop* and the *two-legged jump*, where left and right legs receive the same pair of signals, and signals corresponding to muscle groups in one leg are either in phase (*two-legged hop*) or half-period out of phase (*two-legged jump*).

The *two-legged jump* is identified with the periodic solution $(x_{LL},x^S_{LL},x_{LL},x^S_{LL})$ of system (1), that has symmetry pairs $(H,K) = (\Gamma_{\text{biped-robot}},\mathbf{Z}_2(\kappa))$.

Let $X(t) = (x_{LL_1},x_{RL_1},x_{LL_2},x_{RL_2})$ be a periodic solution of (1). Permutation κ switches signals sent to left and right legs. Applying κ to the *two-legged jump* does not change that gait since LL_i and RL_i, $i = 1,2$ receive the same set of signals. The permutation κ is called a *spatial* symmetry for the *two-legged jump*. The periodic solution $X(t)$ after applying $kappa$ is changed to

$$\tilde{X}(t) = (x_{LL_1},x_{LR_1},x_{LL_1},x_{RL_1})$$

Symmetry $\Gamma_{\text{biped-robot}}$ forces the signals sent to the two cells in each leg to be the same, up to a phase shift of $1/2$. Then, the final periodic solution identified with the *two-legged jump* is:

$$\left(x_{LL}, x^S_{LL}, x_{LL}, x^S_{LL}\right)$$

The *two-legged hop* is identified with the periodic solution $(x_{LL}, x_{LL}, x_{LL}, x_{LL})$ of system (1), with symmetry pairs $(H,K) = (\Gamma_{\text{biped-robot}}, \Gamma_{\text{biped-robot}})$.

Let $X(t) = (x_{LL_1}, x_{LR_1}, x_{LL_2}, x_{RL_2})$ be a periodic solution produced by (1). The spatial permutation $\Gamma_{\text{biped-robot}}$ switches signals sent to the four oscillators. The periodic solution $X(t)$ after applying $\Gamma_{\text{biped-robot}}$ is changed to

$$\tilde{X}(t) = (x_{LL_1}, x_{LL_1}, x_{LL_1}, x_{LL_1})$$

As all oscillators receive the same set of signals, there is no room for spatiotemporal symmetry. Then, the final periodic solution identified with the *two-legged hop* is:

$$(x_{LL}, x_{LL}, x_{LL}, x_{LL})$$

2.4 Bifurcation of bipedal gaits

Suppose that $x(t)$ is a hyperbolic periodic solution to CPG biped-robot, corresponding to a gait with spatiotemporal symmetries H and spatial symmetries K, where $K \subset H \subset \Gamma_{\text{biped-robot}}$. Theorem 3.6 (page 67) of Golubitsky and Stewart [11] implies that system (1) may support a hyperbolic stable periodic solution, with spatial symmetry K and spatiotemporal symmetry H. Thus, in the case where the discrete part is inserted in all limbs and with equal value, periodic solutions produced by the CPG model biped-robot are stable and are identified with the bipedal gaits in Table 1 and Table 1 [26].

The identification of periodic solutions in Table 1 may also be done using bifurcation theory. To compute stability of those periodic solutions, it is necessary to compute the eigenvalues of the linearization L of system (1) at an equilibrium $X = (x,x,x,x)$. If the group action is complicated, bare hands computation or computer algebra systems give little insight into the results.

The $\Gamma_{\text{biped-robot}}$ symmetry group of system (1) implies that L commutes with $\Gamma_{\text{biped-robot}}$, and this translates in nice properties on matrix L. We can decompose $\mathbf{P} = (\mathbf{R}^k)^4$ into a direct sum of $\Gamma_{\text{biped-robot}}$ irreducible subspaces. In general, this decomposition is not unique, nevertheless, if we use components that combine together all of the $\Gamma_{\text{biped-robot}}$ irreducible subspaces that lie in a fixed isomorphism class, then we obtain a decomposition that is unique. This decomposition is also invariant under L.

Let V_{jk} be the sum of all irreducible subspaces that are isomorphic to a representation $\lambda \in \Gamma_{\text{biped-robot}}$. That is, V_{jk} is the isotypic component of $(\mathbf{R}^k)^4$, corresponding to λ. Subspaces V_{jk} are defined as follows. Let $\sigma = e^{i\pi/2}$, then V_{jk} is spanned, over $\mathbf{C}$, by:

$$v_{jk} = \left(1, (-1)^k; \sigma^j, (-1)^k\sigma^j; \sigma^{2j}, (-1)^k\sigma^{2j}; \sigma^{3j}, (-1)^k\sigma^{3j}\right)$$

Since $\Gamma_{\text{biped-robot}}$ is abelian, there are four distinct one-dimensional representations of $\Gamma_{\text{biped-robot}}$. These representations are distinguished by their kernels and are denoted by such. The decomposition of $\mathbf{P}$ into irreducibles is into the representations P_{jk}, where

$$P_{jk} = \begin{cases} \mathrm{Re}(V_{jk}) & \text{if } j = 0,2 \\ \mathrm{Re}(V_{jk} \oplus V_{3-j,k}) & \text{if } j = 1 \end{cases}$$

See Table 2 for a correspondence between gaits in Table 1 and irreducibles P_{jk}.

The isotypic decomposition can be used to block-diagonalize L (see Theorem 2.12 in [11]). The later makes easier the task of computing the eigenvalues of L. Let A, B, C, D be $k \times k$ matrices, where A is the

P_{jk}	Bipedal gaits
P_{00}	*two-legged hop*
P_{31}	*run*
P_{01}	*two-legged jump*
P_{20}	*walk*

Table 2: Periodic solutions of CPG biped-robot and corresponding isotypic components.

part of the Jacobian L corresponding to the oscillators' internal dynamics, B corresponds to the bilateral coupling, C is the ipsilateral coupling, and D to the cross coupling. Matrix L is given by:

$$L = \begin{bmatrix} A & B & C & D \\ B & A & D & C \\ C & D & A & B \\ D & C & B & A \end{bmatrix}$$

It follows from Table 2 that the eigenvalues of L are the eigenvalues of block matrices $L_{\Gamma_{\mathsf{biped-robot}}}$, $L_{\mathbf{Z}_2(\omega)}$, $L_{\mathbf{Z}_2(\kappa)}$, $L_{\mathbf{D}_3(\kappa\omega)}$. See (2).

$$L_{\Gamma_{\mathsf{biped-robot}}} = A+B+C+D, \quad L_{\mathbf{Z}_2(\kappa)} = A-B+C-D, \quad L_{\mathbf{Z}_2(\omega)} = A+B-C-D, \quad L_{\mathbf{Z}_2(\omega\kappa)} = A-B-C+D \tag{2}$$

As $\Gamma_{\mathsf{biped-robot}}$ is an abelian group, generically, Hopf bifurcation occurs with simple pairs of purely imaginary eigenvalues. Thus, we may compute Hopf bifurcation points for each matrix L_λ and stability of the corresponding periodic orbits. Table 3 shows the correspondence between periodic orbits obtained from L_λ and bipedal gaits in Table 1. For more information see [12, 26].

L_λ	Bipedal gaits
$L_{\Gamma_{\mathsf{biped-robot}}}$	*two-legged hop*
$L_{\mathbf{Z}_2(\omega)}$	*run*
$L_{\mathbf{Z}_2(\kappa)}$	*two-legged jump*
$L_{\mathbf{D}_2(\omega\kappa)}$	*walk*

Table 3: Periodic solutions of CPG biped-robot obtained from Hopf bifurcation of L_λ.

In all other cases, where the discrete part is inserted in all limbs with unequal values, or is only inserted in some of the limbs, the periodic solution obtained is not any of the bipedal gaits in Table 3. This is explained by the fact that the symmetry group of this solution $\tilde{x}(t)$ is no longer $\Gamma_{\mathsf{biped-robot}}$, but is a smaller group. This, the solution $\tilde{x}(t)$ might be 'close' to $x(t)$ but is not the same.

Mathematically, this can be explained as follows. Suppose that we consider a small forced symmetry-breaking of the equations (1), so that there are two distinct functions in (1) modeling the oscillators' internal dynamics. Hyperbolicity implies that a solution $x(t)$ of CPG biped-robot perturbs to a periodic solution of a network close to biped-robot, but with oscillators of different types. For instance, you can consider left oscillators different from right oscillators, so you have function F_1 modeling the dynamics of cells on the left, and function F_2 modeling the dynamics of cells on the right, with $F_1 \neq F_2$.

Let's call this new network biped-robot-2. The symmetry group of biped-robot-2 is $\Gamma_{\mathsf{biped-robot-2}} = \mathbf{Z}_2(\omega) \subset \Gamma_{\mathsf{biped-robot}}$. The perturbed solution $\tilde{X}(t)$ has symmetry groups $H' = H \cap \Gamma_{\mathsf{biped-robot-2}}$ and $K' = K \cap \Gamma_{\mathsf{biped-robot-2}}$. These points are verified as follows. Uniqueness of the perturbed periodic solution implies that all symmetries in $K \cap \Gamma_{\mathsf{biped-robot-2}}$ fix the perturbed trajectory pointwise since the perturbed equations have $\Gamma_{\text{biped-robot-2}}$-equivariance, that is, they have $\Gamma_{\text{biped-robot-2}}$ symmetry group. So $K \cap \Gamma_{\text{biped-robot-2}} \subset K'$. Conversely, any symmetry in $K' \subset \Gamma_{\text{biped-robot-2}}$ must be in K again by the uniqueness of hyperbolic periodic solutions in a small neighborhood. An analogous argument is valid for H'.

This raises some questions concerning applicability of these 'close' solutions to motor actuators in robots. How small should be $\|x(t) - \tilde{x}(t)\|$ so that these differences are forgotten by the physical mechanism? Considering consumer and educational robots, particularly considering prototype and research oriented devices, presently these apply "hobby "servos which include digital microcontrollers. Specifically, Dynamixel servos from Robotis, appear as an attractive and appealing type of servos, considering feasibility for precision control and dynamic operation. The capabilities of these servos, were extensively analyzed in [32]. Their tests have shown that the average uncalibrated servo has a position bias within $\pm 0.21^o$ at the ends of its range of motion, and that, when unloaded, the overall standard deviation of position feedback from target position is 0.36^o. These tests have demonstrated that the servo accurately reaches its target position when unloaded.

Therefore, considering solutions that are close to the ideal solution, with an error smaller than 10^{-3} or 10^{-2}, these considerations are not relevant for physical, real devices that employ these servos or alike.

2.5 Stability of bipedal gaits

Stability of the periodic solutions is computed for each L_λ matrix, see (3). Consider the Hopf oscillator [18] as cells' internal dynamics in CPG (1). Let HH be the linearization of the single Hopf oscillator and matrix Z the following matrix

$$Z = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$$

Matrices (2) can be written, for synaptic coupling, as:

$$\begin{array}{ll} L_{\Gamma_{\text{biped-robot}}} = HH - (k_1 + k_2 + k_3)Z & L_{\mathbf{Z}_2(\kappa)} = HH - (k_1 - k_2 + k_3)Z \\ L_{\mathbf{Z}_2(\omega)} = HH - (-k_1 + k_2 - k_3)Z & L_{\mathbf{Z}_2(\omega\kappa)} = HH - (k_1 + k_2 - k_3)Z \end{array} \quad (3)$$

where $A = HH$, $B = -k_1 Z$, $C = -k_2 Z$ and $D = -k_3 Z$.

The eigenvalues of $HH - (k_i \pm k_j \pm k_l)$, for $i \neq j \neq k$, can be computed using, for instance XPPAUT [8]. The *two-legged hop* appears when $k = k_1 + k_2 + k_3$ crosses a bifurcation value, analogously for *walk*, that is obtained when parameter $k = k_1 + k_2 - k_3$ crosses another bifurcation value. Additionally, we need the remaining eigenvalues to be negative.

3 Numerical Simulations

We simulate the CPG model biped-robot. In each CPG-unit, the discrete part $y(t)$ is inserted as an offset of the rhythmic part $x(t)$. The coupling is either diffusive or synaptic.

The equations for the rhythmic motor primitive are known as the modified Hopf oscillators [18] and are given by:

$$\begin{array}{rcl} \dot{x} & = & \alpha(\mu - r^2)x - \omega z = f(x,z) \\ \dot{z} & = & \alpha(\mu - r^2)z + \omega x = g(x,z) \end{array} \quad (4)$$

where $r^2 = x^2 + z^2$, $\sqrt{\mu}$ is the amplitude of the oscillation. For $\mu < 0$ the oscillator is at a stationary state, and for $\mu > 0$ the oscillator is at a limit cycle. At $\mu = 0$ it occurs a Hopf bifurcation. Parameter ω is the intrinsic frequency of the oscillator, α controls the speed of convergence to the limit cycle. ω_{swing} and ω_{stance} are the frequencies of the swing and stance phases, $\omega(z) = \frac{\omega_{stance}}{\exp(-az)+1} + \frac{\omega_{swing}}{\exp(az)+1}$ is the intrinsic frequency of the oscillator. With this ODE system, we can explicitly control the ascending and descending phases of the oscillations as well as their amplitudes, by just varying parameters ω_{stance}, ω_{swing} and μ. These equations have been used to model robots' trajectories [7, 19].

The system of ordinary differential equations that models the discrete primitive is the VITE model given by [3]:

$$\begin{aligned} \dot{v} &= \delta(T - y - v) \\ \dot{y} &= G\, max(0, v) \end{aligned} \tag{5}$$

This set of differential equations generates a trajectory converging to the target position T, at a speed determined by the difference vector $T - y$, where y models the muscle length, and G is the go command. δ is a constant controlling the rate of convergence of the auxiliary variable v. This discrete primitive controls a synergy of muscles so that the limb moves to a desired end state, given a volitional target position.

The coupled systems of ODEs that model CPG biped-robot where the discrete part is inserted as an offset of the rhythmic primitive, for synaptic and diffusive couplings, are given by:

$$\begin{aligned} \dot{x}_i &= f_1(x_i, z_i) + k_1 h(x_{i+1}, x_i) + k_2 h(x_{i+2}, x_i) + k_3 h(x_{i+3}, x_i) \\ \dot{z}_i &= g_1(x_i, z_i) + k_1 h(z_{i+1}, z_i) + k_2 h(z_{i+2}, z_i) + k_3 h(z_{i+3}, z_i) \end{aligned} \tag{6}$$

where $f_1(x_i, z_i, y_i) = \alpha(\mu - r_i^2)x_i - \omega z_i$, $g_1(x_i, z_i, y_i) = \alpha(\mu - r_i^2)z + \omega x_i$, and $r_i^2 = (x_i - y_i)^2 + z_i^2$. k_i are the coupling strengths. Indices are taken modulo 4. Functions $h(z_j, z_i)$, represents synaptic coupling when written in the form $h(z_j, z_i) = z_j$, and diffusive coupling when written as $h(z_j, z_i) = z_j - z_i$.

Parameter values used in the simulations are $\mu = 10.0$, $\alpha = 5$, $\omega_{\text{stance}} = 6.2832$ rads^{-1}, $\omega_{\text{swing}} = 6.2832$ rads^{-1}, $a = 50.0$, $G = 1.0$, $\delta = 10.0$.

We vary $T \in [0, 25]$, in steps of 0.1, for the periodic solution, identified with the *run*. We start from a stable purely rhythmic periodic solution. Then, we vary T, and simulate the periodic solution, with this new offset, until a new stable solution is found. In the case of a periodic solution, we verify that it is still the *run*, and we compute its amplitude and frequency values, that are then plotted. Values of the offset T such that the achieved stable solution is an equilibrium are not plotted.

3.1 Equal values of the discrete primitives

In this section, we consider that all oscillators are perturbed by the same value of the discrete part. We follow the procedure mentioned above for the numerical simulations.

We compute the amplitude and frequency values of the periodic solutions produced by CPG biped-robot, and identified with the bipedal *run*, for different values of the offset T. We find that they do not vary with the superimposition of the discrete to the rhythmic primitives. Therefore, it is possible to use them for generating trajectories for the joint values of real biped robots, since varying the joint offset will not affect the required amplitude and frequency of the resultant trajectory, nor the gait.

3.2 Different values of the discrete primitives

In this section, we consider different discrete values as offsets of the rhythmic primitives. Simulations test different offsets in left and right legs, $T_{LL} \neq T_{RL}$, with the subcase of only one of the legs being perturbed by the offset. We then follow the procedure mentioned above, to obtain a stable periodic solution, after insertion of the discrete part.

In these cases, the periodic solution, obtained after the insertion of the discrete parts, is not a solution identified with the biped *run*, since amplitude and frequency values of the signals of each oscillator differ by an amount $\geq 10^{-3}$. When $\|T_{LL} - T_{RL}\| \leq 1$ then the differences in amplitude and frequency values are lesser or equal than 10^{-3}.

Here, the questions raised in the end of subsection "Bifurcation of bipedal gaits" are valid again. Mathematically, these solutions are no longer identified with any of the primary gaits in Table 1. But from a Robotics point of view, solutions that are close to the ideal solution, with an error smaller than 10^{-3} or 10^{-2}, may be applied to servos of physical, real devices [32].

3.3 Different forms of inserting the discrete primitives

We have also tested, numerically, a new form of inserting the primitive part in the rhythmic primitive. The offset is summed to the output of the rhythmic part. That is, the signal that would be applied to servos of physical devices is of the form $\tilde{x}(t) = x(t) + y(t)$, where $x(t)$ is the rhythmic signal and $y(t)$ is the offset of the discrete part.

We followed the same procedure described above, to obtain a stable periodic solution, after insertion of the discrete part. We considered (a) equal discrete primitives in the two legs, and (b) two distinct primitives for the two legs. In the two cases, the amplitude and frequency of the periodic solution, identified with the *run*, remain constant, for variation of the offset. This form of introducing the discrete part in the purely rhythmic movement is also a valid one, when considering implementations of the proposed controllers for generating trajectories for the joints of real biped robots.

4 Conclusion

We study the effect on the periodic solutions produced by a CPG model for biped robots movements of superimposing two motor primitives: discrete and rhythmic. These periodic solutions are identified with the bipedal *run*. The CPG model biped-robot has the same architecture as a CPG model for biped animals, developed in [26]. There is, however, an important distinction: here, each neuron/cell (CPG-unit) combines two motor primitives, discrete and rhythmic. We simulate the CPG model biped-robot considering the discrete primitive as an offset of the rhythmic primitive, and two distinct coupling functions. We compute the amplitude and the frequency values of the periodic solutions identified with the *run*, for values of the discrete primitive target parameter $T \in [0, 25]$. We consider two cases, where oscillators of the two legs receive the same offset and where offsets of the two legs are different. Numerical results show that, for the same offset in all oscillators, amplitude and frequency values are almost constant, for both couplings. For different offsets in the two legs, amplitude and frequency values of the solutions differ by an amount $\geq 10^{-3}$. These solutions, however, may be used in real physical devices, since errors of magnitude 10^{-3} or 10^{-2} are negligible. These results may be generalized for robots with a smaller or larger number of legs, since the CPG model used here easily extrapolates for $2n$ legs. Future work includes the development a biped robot experiment, in which these findings may be replicated.

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